# Time-Dependent Price Adjustment and the Neutrality of Money

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# Motivation

- Empirical evidence suggests in a narrowly-defined market,
  - Failure of *"the law of one price"* - persistent price dispersion
- Introducing costly consumer search can rationalize price dispersion (Burdett and Judd,83; Stahl, 89).
- Macroeconomists are interested in price dispersion for a diff reason.
- This persistent component of the price dispersion is treated as independent of transitory one related to inflation.
  - This paper: integrating both is essential to understand monetary policy transmission.
- Research questions: how does the consumer search behavior affect the monetary policy transmission?

# Main Results

1. Monetary neutrality result: monetary non-neutrality based on Calvo does not survive a small perturbation on consumer search behavior.
    - Robust to adding sequential search
2. Provide search-theoretic foundation for aggregate price stickiness and the sufficient statistics that characterize the inflation dynamics.
    - Menu cost perturbation and equilibrium strategy purification
    - Large shocks; asymmetry; robustness (If time permits)

# A Simple Search Model (BJ83)

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- Continuum of firms sell a homog good to continuum of consumers.
  - Firms first move and each firm sets a single price.
  - Consumers know $F(p)$ and choose the integer number of firms, $n$, to sample at a cost of c per firm.
- Consumers mix. Denote $\{q_1, q_2, ..., q_n\}, \forall n$. We call $q_1$ consumers captive; o/w, non-captive.
- Firms mix between any price on a connected support. The price distribution is denoted by $F(p)$.
- In one equilibrium, $\{F(p), \{q_k\}_{k=1}^{\infty}\}$ has the following property,
  - $0 < q_1 = q < 1$ and $q_2 = 1 - q$
  - Firms set $F(p)$ to make consumers indifferent b/w sampling 1 and 2.
  - Consumers set $q$ to make firms indifferent b/w any price in the support of $F(p)$.

# Solve the Distribution

- Firms optimally choose a mixed strategy of pricing,

$$\Pi = D(Q)(Q - r)q = D(p)(p - r)(q + 2(1 - q)(1 - F(p)))$$

  Tradeoff: less market share v.s. higher surplus per consumer.
- $\pi(p) = D(p)(p - r)$ is continuous and increasing.
- Solve the distribution:

$$F(p) = 1 - \frac{q}{2(1-q)}\left(\frac{\pi(Q)}{\pi(p)} - 1\right), \ \forall b \leq p \leq Q$$

- In the following, I will take q as the primitive. But imagine a mapping from c to q in the background.

# Price Distribution

$D(p) = p^{-\sigma}$ and $r = 1$; $Q = 1.7$

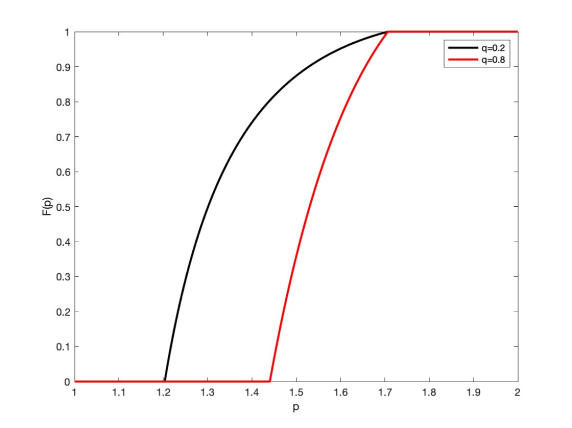


**Figure 1:** price distribution for different $q$

# Calvo's Monetary Non-neutrality Result

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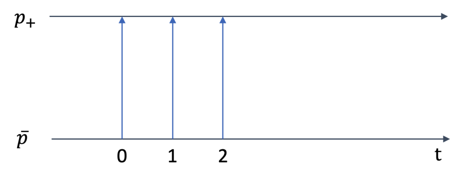
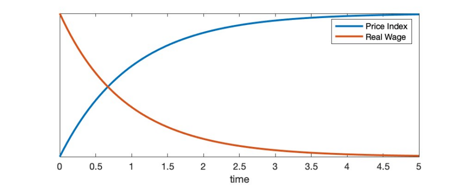
- Suppose CB announces and controls nominal wages $\{W_t\}_{t=0}^{\infty}$ with $|log W_t - log \bar{W}| < \bar{\delta}$. We define $\hat{X}_t = log X_t - log \bar{X}$.
- Firms' marginal cost is nominal wage and has a single optimal price.
- Suppose nominal wage increases by 1% permanently by central bank.
- If pricing is flexible, all firms increase their price by 1% simultaneously.
- No change in the real wage!
- Change the unit of money won't have real effect.

- Calvo assumes only a fraction of firms $\lambda \in (0, 1)$ can adjust prices.




- Real effect of monetary intervention is on the same magnitude of monetary shock itself!

# Main Result: Monetary Neutrality Result

# Monetary Neutrality Result

We consider a small perturbation, i.e., $q_1 = q,\ q_2 = 1 - q$.
**Assumption 1:** every period, consumers re-sample the firms.

**Monetary Neutrality Theorem**

Under assumption 1, for every $\bar{\delta} > 0$ and any nominal wage sequence $\{W_t\}$ of which $|\hat{W}_t| < \bar{\delta}$, there exists $\bar{q}_2 \in (0, 1)$ such that for every $q_2 > \bar{q}_2$ and every $t$, $\hat{\omega}_t = O(\delta^2)$.

- Small monetary shock has only second-order effect on real wage.
    - 1% monetary shock implies the magnitude of 0.01% of real effect.
- Calvo's result is a knife edge case when $q = 1$.

# Case: Inflation

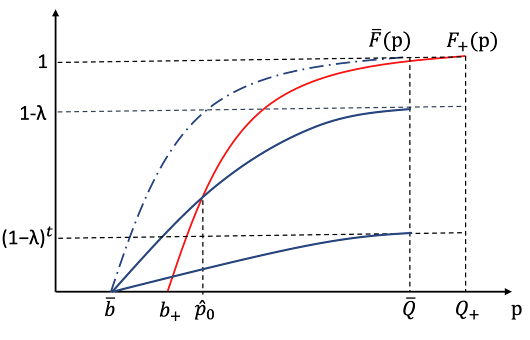
Consider $\{W_t\}_{t=0}^{\infty} = \{\bar{W}|W_+, W_+, W_+, ...\}$ and $\{\hat{Q}_t\}_{t=0}^{\infty} = \{\hat{W}_t\}_{t=0}^{\infty} = \delta$.

**Proposition: Law of Motion of Price Distribution**

The price distribution evolves as the following,

$$H_t(p) = \begin{cases} F_+(p) & \text{for all } p \in [\hat{p}_t, Q_+] \\ (1-\lambda)^t \bar{F}(p) & \text{for all } p \in [b, \hat{p}_t] \end{cases}$$

$\hat{p}_t$ is determined by $F_t(p) = (1-\lambda)^t \bar{F}(p)$ and $\hat{p}_t \to b_+$.



# Case: Inflation

- The price distribution faced by a typical consumer is defined as

$$G_t(p) = qH_t(p) + (1-q)(1-(1-H_t(p)^2))$$

- The price index is

$$P_t = \int_b^{Q_+} pdG_t(p) = \int_b^{Q_+} pdH_t(p)$$

  Denote the efficient price index

$$P_+ = \int_{b_+}^{Q_+} pdF_+(p) = \delta$$

- $P_t - P_+ = -\frac{1}{2}(b_+ - b)F(\hat{p}_t) = O(\delta^2) \Rightarrow \hat{P}_t = \delta + O(\delta^2) \Rightarrow \hat{\omega}_t = O(\delta^2)$

**Proposition**

Fix a nominal wage path $\{W_t\}_{t=0}^{\infty} = \{\bar{W} | W_+, W_+, W_+, ...\}$ and $\hat{W}_+ = \delta$, given $q$, for $\delta \to 0$, the change in real wage in on the magnitude of $\delta^2$.

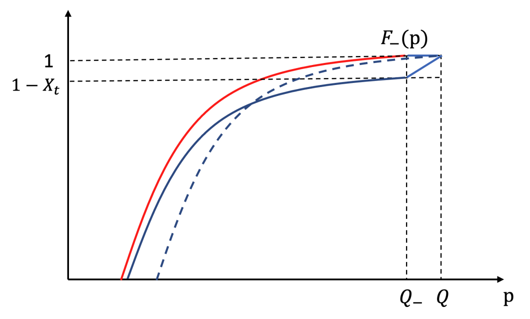
# Case: Deflation

Consider $\{W_t\}_{t=0}^{\infty} = \{\bar{W} | W_-, W_-, W_-, ...\}$ and $\{\hat{Q}_t\}_{t=0}^{\infty} = \{\hat{W}_t\}_{t=0}^{\infty}$.

**Proposition: Law of Motion of Price Distribution**

The price distribution evolves as the following,

$$H_t(p) = \begin{cases} F_-(p) - (1 - \bar{F}(Q_-))(1-\lambda)^t \frac{\pi(Q_-)}{\pi(p)} & \text{for all } p \in [b_t, Q_-] \\ 1 - (1 - \bar{F}(p))(1-\lambda)^t & \text{for all } p \in [Q_-, Q] \end{cases}$$



# Case: Deflation

- Denote the efficient price index

$$P_- = \int_{b_-}^{Q_-} p dF_-(p) = -\delta$$

- $P_t = (1 - \bar{F}(Q_-))(1-\lambda)^t \frac{2(1-q)}{q} P_- + P_-$
- $(1 - \bar{F}(Q_-))P_- = f(Q_-)(Q - Q_-)P_- = O(\delta^2)$

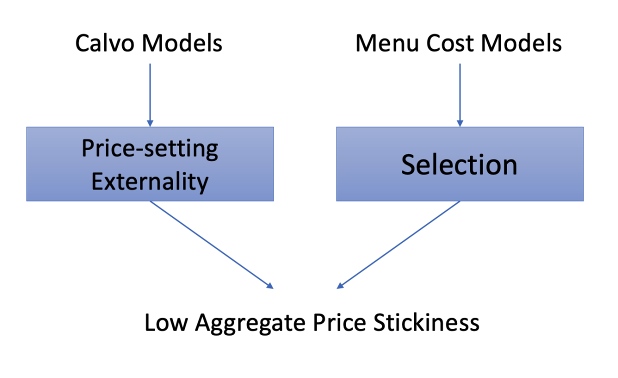
**Proposition**

Fix a nominal wage path $\{W_t\}_{t=0}^{\infty} = \{\bar{W}|W_-, W_-, W_-, ...\}$ and $\hat{W}_- = -\delta$, given $q$, for $\delta \to 0$, if $\pi(p)$ is increasing up to a single peak, the change in real wage in on the magnitude of $\delta^2$.

▸ Step 2&3

# Remark

- Price-setting externality: a small fraction of optimizing firms is enough to make majority of firms optimal.



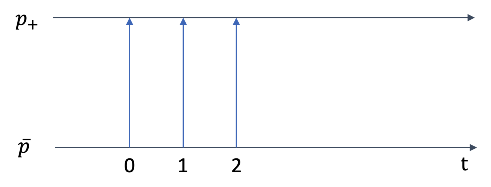

- Note: The result does not mean that NK mechanism is not true. But the micro-foundation for price stickiness is not robust.

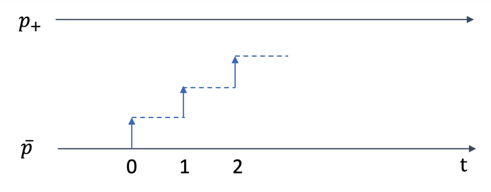
# Sequential Search and Robustness of Main Result

# Sequential Search and Monetary Policy

- Until now, simultaneous search with fixed price count distribution.
- How about we add sequential search instead of simultaneous one?
    - stochastic price counts
    - keep searching until getting a price quote lower than $R$.

# Sequential Search and Monetary Policy



# Apply Same Perturbation

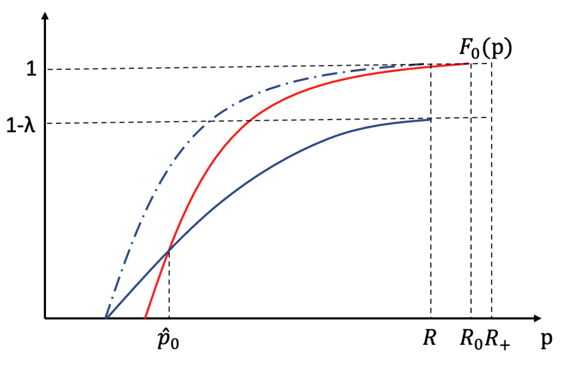
- Is this intuition robust? Let's apply the same perturbation.
- $\mu$: simultaneous searchers; $q\mu$: search once ; $(1-q)\mu$: search twice; $1-\mu$: sequential searcher.

**Monetary Neutrality Theorem with Sequential Search**

Under Assumption 1, for every $\bar{\delta} > 0$ and $t$ and every nominal wage sequence of which $|\hat{W}_t| < \bar{\delta}$, there exists $\bar{q} \in (0,1)$ such that for every $q < \underline{q}$ and $t$, $\hat{\omega}_t = O(\delta^2)$.

- We need enough sequential searchers.
- Reservation price is determined by $\int_b^{R_t} \left( \int_p^{R_t} D(p)dp \right) dH_t(p) = c$
- $\int_b^{R_t} \left( \int_p^{R_t} D(p)dp \right) dH_t(p) - \int_b^{R_+} \left( \int_p^{R_+} D(p)dp \right) dF_+(p) = O(\delta^2)$

## Step 1 Case 1




$$\frac{\int_{\bar{b}}^{\hat{p}_0}(1-\lambda)\bar{F}(p)D(p)dp + \int_{\hat{p}_0}^{R_0}F_+(p)D(p)dp}{(R_0-b_0)D(R_0)} = c$$

- Left-tail difference is second-order. $R_0 = R_+ + O(\delta^2)$

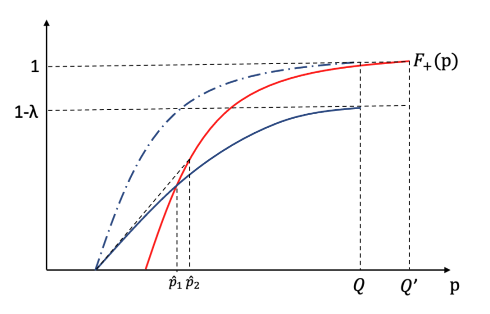
# Remark

- The intuition that extra strategic complementarity of sequential search would induce more price stickiness is not robust.
- The left tail of $H_t(p)$ is the key to determination of reservation price. However, price-setting externality makes the left tail small.
- Our main result is robust to two prominent kinds of consumer search behavior in the literature.

# Step 2 Case 1

For nondecreasing sequence of nominal wage, i.e., monetary expansion, $\{W_t\}_{t=0}^{\infty} = \{\bar{W} | W_+, W'_+, W'_+, ...\}$ and $W'_+ > W_+$.

- Firms know only the left tail can be non-optimal at $t = 1$. (selection)

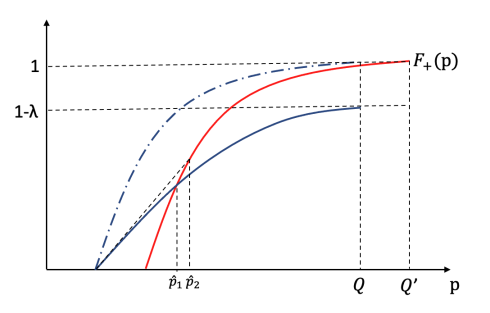



Go Back

## Step 2 Case 1

For nondecreasing sequence of nominal wage, i.e., monetary expansion, $\{W_t\}_{t=0}^{\infty} = \{\bar{W} | W_+, W'_+, W'_+, ...\}$ and $W'_+ > W_+$.

- Firms know only the left tail can be non-optimal at $t = 1$. (selection)
- The price distribution on left tail is complex, but the price index distortion is bounded by $P_0 - P_+ \leq \frac{1}{2}(\hat{p}_2 - b)F_+(\hat{p}_2)$.



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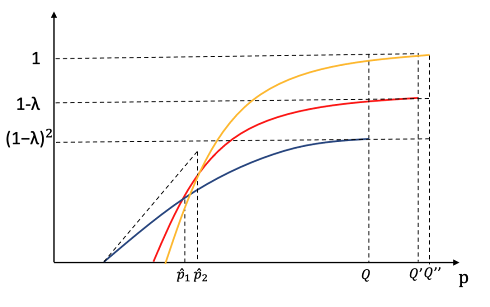

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For nonincreasing sequence of nominal wage, i.e., monetary expansion, $\{W_t\}_{t=0}^{\infty} = \{\bar{W} | W_+, W'_+, W'_+, ...\}$ and $W'_+ > W_+$.

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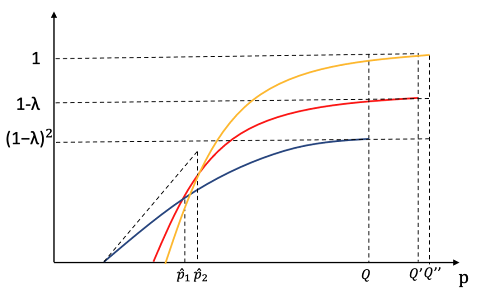

# Step 2 Case 2

For nondecreasing sequence of nominal wage, i.e., monetary expansion, $\{W_t\}_{t=0}^{\infty} = \{\bar{W} | W_-, W'_-, W'_-, ...\}$ and $W'_- < W_-$.

- Firms know only the right tail can be non-optimal at $t = 1$.
- The price index distortion is bounded by $P_0 - P_- = X_0 \frac{2(1-q)}{q} P_- + O(\delta^3)$.